Doctoral Program in Economics

UNIVERSITÀ DEGLI STUDI FIRENZE | UNIVERSITÀ DI SIENA | UNIVERSITÀ DI PISA

Academic year 2026/27

MATHEMATICS

Period:
First term

Course hours: 20

Teachers:

A. Caravaggio (10h), P. Mazzarisi (10h)

Exam methods:
written exam

Prerequisites:

- Algebra: vectors and matrices, operations on vector spaces, linear dependence among vectors, rank and determinant of a matrix, linear systems.
- One-variable calculus: domains, limits, continuity and differentiability, convexity, integral calculus (both indefinite and definite integrals).
- Multivariable calculus: partial derivatives, gradient vector, chain rule.

Module I (P. Mazzarisi) Program

Topics in Linear Algebra: Determinant and its geometric interpretation. Partitioned matrices and their inverses. Basic notions on abstract spaces. Main results on linear systems. Eigenvalues and eigenvectors. Diagonalization. Quadratic forms and quadratic forms with linear constraints.

Topics in Multivariable Calculus: Balls, open sets, closed sets, bounded sets and compact sets in Euclidean spaces. Generalities on functions of several variables. Continuity. Partial derivatives and gradient. Stationary points. Second partial derivatives. Schwartz theorem. Hessian matrix. Convex sets. Concave and convex functions. Strictly concave and strictly convex functions. Necessary and sufficient conditions for concavity (convexity). Sufficient conditions for strict concavity (strict convexity). Vector valued functions. Jacobian matrix. Implicit function theorem. Maximum points of a function on a set. General formulation of a maximization problem. Extreme value theorem. Unconstrained maximization problems: necessary first-order conditions for optimality; necessary second-order conditions for optimality; sufficient first-order conditions for optimality. Maximization problems with equality constraints and maximization problems with inequality constraints: Lagrange function; necessary first-order conditions for optimality; sufficient first-order conditions for optimality.

Educational objectives

Methods and techniques covering multivariable calculus and static optimization

Bibliographical references

Sydsæter, K., Hammond, P., Seierstad, A., & Strom, A. (2008). Further mathematics for economic analysis. Pearson education. (Chapters 1, 2 and 3)

Module II (A. Caravaggio) Program

Topics in Continuous Time Dynamic Systems: Introduction to differential equations (first and second order) and their solution. Introduction to dynamic systems (from one-dimensional to n-dimensional). Qualitative analysis on existence and stability of equilibria for 1-D nonlinear dynamic systems. Local Bifurcations in 1-D nonlinear dynamic systems. Qualitative analysis on existence and stability of equilibria for 2-D nonlinear dynamic systems. Local Bifurcations in 2-D nonlinear dynamic systems. Periodic solutions and limit cycles. Applications to economic models.
Topics in Discrete Time Dynamic Systems: Introduction to difference equations (first and second order) and their solution. Introduction to dynamic systems (from one-dimensional to n-dimensional). Qualitative analysis on existence and stability of fixed points for 1-D nonlinear dynamic systems. Local Bifurcations in 1-D nonlinear dynamic systems. Basins of attraction. Qualitative analysis for 2-D nonlinear dynamic systems. Dynamic systems represented by noninvertible maps. Applications to economic models.
Topics in Continuous Time Optimal Control: Introduction to optimal control problems. Optimality conditions for simple problems. Introduction to costate variables and their economic meaning. Problems with fixed final time and terminal condition on the control variable.

Educational objectives

Methods and techniques covering dynamic systems and optimal control.

Bibliographical references

Bischi, G. I., Panchuk, A., & Radi, D. (2016). Qualitative theory of dynamical systems, tools and applications for economic modelling (pages 1-159).

Gandolfo, G. (2009). Economic Dynamics (fourth edition).

Additional information

Problem sets

We will send you 5 problem sets. The solutions will be provided and partly discussed in class.
Problem Set Rules:

- You are expected to submit by email your solutions by the deadline.

- You will organize into groups, as uniform in size as possible (ideally 3-4 persons per group). Each group will hand in one set of solutions.
- We encourage group composition to vary across problem sets. Two people can hand in together no more than two problem sets.
- Solutions must be typewritten (not handwritten).
- Because of time constraints, during the session we shall only discuss the solution to some of the exercises in the problem set.
- Handing in solutions is compulsory. We will use your solutions to compensate for exercises that you miss in the exam. Example: if you miss one topic in the exam, we will check how you did the corresponding problem set, before grading the exam.
- It is better for your learning if you attempt at providing a solution of your own creation rather than copying solutions from other sources.

Exam

A written 3-hour open-book exam concludes the course.

Exam date: To be announced (roughly, just before Christmas break)